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FOUNDATIONS OF THEORY STEPED CYLINDERICAL OPTICAL FIBERS

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Abstract. *The rapid introduction of fiber-optic communication lines into telecommunication networks in recent years has led to significant advantages arising from the peculiarities of the propagation of high-speed radio signals in fiber. For a deep understanding of the processes of signal transmission in fiber-optic communication lines, first of all, it is necessary to know the propagation of electromagnetic waves in fiber-optic communication lines, as well as the exact solution of electrodynamics problems.*

The author analyzes the propagation of hybrid electromagnetic waves in an optical fiber and defines the principles and algorithms of the engineering method for calculating the optical parameters of fibers.

In the work, which consists of two parts, in the first part, the boundary value problem on the propagation of electromagnetic waves in a regular confocal cylindrical stepped fiber is solved, as a result of which: an equation of eigenvalues (EVE) is obtained, which determines the hybrid mode of propagation of electromagnetic waves, in addition, the obtained EVE is compared with similar equations obtained by other authors at the same time, the comparison shows errors or typos in their works. In the second part of the work, the analysis of the transcendental eigenvalue equation for hybrid electromagnetic waves (modes) is carried out and the critical frequencies for the corresponding modes are determined. An engineering method and an algorithm for calculating the main optical parameters of fibers optical communication line (FOCL) are proposed.

Keywords: *wave theory, stepped fiber, electromagnetic waves (EMW), eigenvalue equation (EVE), attenuation coefficient, dispersion, bandwidth.*

ОСНОВИ ТЕОРІЇ СТУПІНЧАТИХ ЦИЛІНДРИЧНИХ ОПТИЧНИХ ВОЛОКОН

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Аннотація. *Стреміке впровадження волоконно-оптичних ліній зв'язку в телекомунікаційні мережі в останні роки призвело до значних переваг, що випливають з особливостей поширення високошвидкісних радіосигналів в оптоволокне. Для глибокого розуміння процесів передачі сигналів в волоконно-оптичних лініях зв'язку, перш за все, необхідно знання поширення електромагнітних хвиль в волоконно-оптичних лініях зв'язку, а також точне рішення задач електродинаміки. Автором здійснюється аналіз поширення гібридних електромагнітних хвиль в оптичному волокні і визначаються принципи та алгоритми інженерного методу розрахунку оптичних параметрів волокон. В роботі, що складається з двох частин, в першій частині вирішується крайова задача про поширення електромагнітних хвиль в регулярному конфокальному циліндричному ступеневу оптичному волокні, в результаті чого: отримано рівняння власних значень (EVE), що визначає гібридний режим поширення електромагнітних, крім того, отримане EVE порівнюється з аналогічними рівняннями, які були отримані іншими авторами, при цьому при порівнянні виявляються помилки або друкарські помилки в їх роботах. У другій частині роботи проводиться аналіз трансцендентного рівняння власних значень для гібридних електромагнітних хвиль (мод) і визначаються критичні частоти для відповідних мод При цьому. знаходяться зв'язку між дискретними значеннями коефіцієнтів поздовжнього поширення сигналу (при вкрай низьких коефіцієнтах затухання) і коефіцієнтами поперечного поширення на частоті робочого сигналу (або на робочій довжині хвилі для заданих параметрів серцевини і оболонки.*

Ключові слова: *хвильова теорія, ступеневу волокно, електромагнітні хвилі (ЕМХ), рівняння власних значень (EVE), коефіцієнт загасання, дисперсія, смуга пропускання.*

Introduction

Impetuous inculcation of fiber optics communication lines in telecommunication networks over the last years resulted in considerable advantages, which followed from peculiarities of high-speed radio-signals propagation in optical fiber. What is necessary for deep understanding of signals transfer processes in optical fiber communication lines first of all is the knowledge of electromagnetic waves propagation in optical fiber lines along with exact solutions of electrodynamics problems. Unfortunately, either until now the stepped fiber optics wave theory solutions in various monographs and textbooks are published with errors or misprinted, that hampers proper estimates of fiber optics parameters and signal transfer regimes. In addition to these reasons, it is advisable to consider the analysis of the propagation of hybrid EMF and, on their basis, to determine the principles and algorithms of the engineering method for calculating the optical parameters of fibers.

Let the infinite optical fiber line be a confocal two-layer cylindric (as shown in Fig. 1(a,b), upon that refraction coefficient of the core is equal to n1, and cladding – n2, d=2a – core diameter, D=2b – cladding diameter. Let the optical fiber line be tied with cylindrical coordinates (Fig. 1a), where er, eφ, ez are unit vectors of cylindrical coordinates. Let us assume that the core radiates (for example, with the help of the semiconductor laser) monochromatic electromagnetic wave (EMW) on ω - frequency, which propagates along the coordinate Z. It is required to determine an electromagnetic field in the core and in cladding, that is to find rojections of electrical and magnetic field vectors in cylindrical coordinates:

$$\vec{E} = E_r\vec{e}_r + E_\varphi\vec{e}_\varphi + E_z\vec{e}_z,\ \vec{H} = H_r\vec{e}_r + H_\varphi\vec{e}_\varphi + H_z\vec{e}_z. \qquad (1)$$

Let the electromagnetic waves propagate in a homogeneous, isotropic and linear dielectric medium $\varepsilon_{1,2}\ (r, \varphi, z,) = const,\ \varepsilon_{1,2}(E) = const,\ \varepsilon_1 > \varepsilon_2$ and let the conductivity of the mediums be equal to zero σ1,2 =0 and their relative magnetic permeability be equal to 1 (μ1,2 = 1).

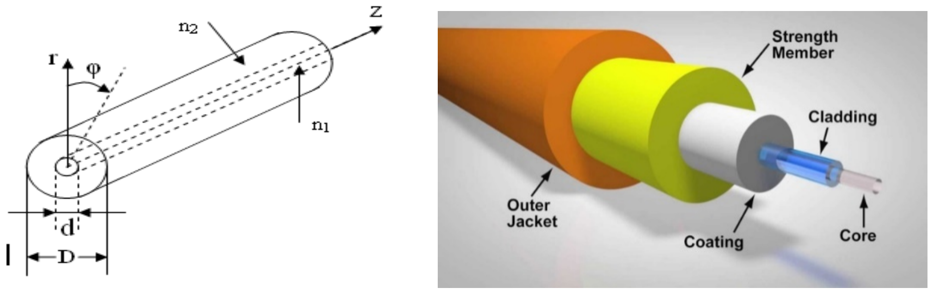

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Fig. 1. (a) Optical fiber line with cylindrical coordinates; (b) Fiber Optical Communication line (FOCL)

Singularities of problem

In any cylindrical wave guide an electromagnetic field can be submitted as Transversal Electric – TEnm – Hnm (at longitudinal component an electrical fields EZ = 0)), Transversal Magnetic - TMnm - Enm (at longitudinal component a magnetic field is equal to zero HZ = 0) and EHnm and HEnm hybrid electromagnetic waves. Let us write the Maxwell's equations for the considered boundary value problem in complex form to solve it:

$$rot\,\vec{\dot{H}} = \mathrm{j}\,\omega\varepsilon\,\vec{\dot{E}}\ ,\ rot\,\vec{\dot{E}} = -j\omega\mu\,\vec{\dot{H}}\ ,\ div\varepsilon\,\vec{\dot{E}} = 0, div\mu\,\vec{\dot{H}} = 0, \qquad (2)$$

at the following boundary conditions:

$$\dot{E}_{z1} = \dot{E}_{z2,}\dot{E}_{\phi1} = \dot{E}_{\phi2,}\ \dot{H}_{z1} = \dot{H}_{z2,}\dot{H}_{\phi1} = \dot{H}_{\phi2,}\ \varepsilon_1\dot{E}_{r1} = \varepsilon_2\dot{E}_{r2,}\dot{H}_{r1} = \dot{H}_{r2,}. \qquad (3)$$

It is admissible, that the complex amplitudes of electrical and magnetic strength field vectors depend on time, as e jωt, and on a longitudinal coordinate z, as e-jkz. Then the Maxwell's equations are transformed into Helmholtz's equations [1-3]:

$$\nabla\vec{\dot{H}} + \mathrm{k}\vec{\dot{H}}=0, \qquad \nabla\vec{\dot{E}} + \mathrm{k}\vec{\dot{E}}=0, \qquad (4)$$

where k2 = ω2εμ is a square of wave number, which characterizes the change of the electromagnetic field (EMF) components along the Z coordinate. Let us seek the longitudinal components of the EMF. Assuming that:

$$\dot{E}\mathrm{z} \sim e^{-\gamma z} \cdot e^{j\omega t},\ \dot{H}\mathrm{z} \sim e^{-\gamma z} \cdot e^{j\omega t}, \qquad (5)$$

for this relatively longitudinal component may be written down as the Helmholtz's equations:

$$\left.\begin{aligned}\frac{\partial^2 \acute{H}_z}{\partial r^2}+\frac{1}{r}\frac{\partial \acute{H}_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \acute{H}_z}{\partial \phi^2}+g^2\acute{H}_z=0\\ \frac{\partial^2 \acute{E}_z}{\partial r^2}+\frac{1}{r}\frac{\partial \acute{E}_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \acute{E}_z}{\partial \phi^2}+g^2\acute{E}_z=0\end{aligned}\right\}, \qquad (6)$$

where g2 1,2 = k2 1,2 + γ2 (8) - transverse propagation coefficients of electromagnetic waves [g1 (in core, along r) or g2 (in cladding, along r)], γ - longitudinal propagation coefficient. After simple transformations, we obtain [1,2]:

$$\left.\begin{aligned}\dot{E}_z(r,\phi,z)=\\ \dot{H}_z(r,\phi,z)=\end{aligned}\right\}[\dot{A}_m\cdot J_m(gr)+\dot{L}_m\cdot N_m(gr)]\cdot cos(m\phi)\cdot exp(-\gamma z), \qquad (7)$$

or

$$\left.\begin{aligned}\acute{E}_z(r)=\\ \acute{H}_z(r)=\end{aligned}\right\}\dot{A}_m\frac{1}{2}\left[H_m^{(2)}(jgr)+H_m^{(1)}(jgr)\right]+j\dot{L}_m\frac{1}{2}\left[H_m^{(2)}(jgr)-H_m^{(1)}(jgr)\right], \qquad (8)$$

where: Jm(gr) and Nm(gr) – Bessel and Neiman functions m - order, Hm(1)(gr) and Hm(2)(gr) – Hankel's functions first and second type m - order, Km and Lm - unknown integral constants, m – number, which characterizes the change of EMF along φ –coordinate.

Coupling equations for stepped fiber optics [1,2] will be as follows:

$$\acute{E}_r=-\frac{1}{g^2}\left(\gamma\frac{\partial \acute{E}_z}{\partial r}+\frac{j\omega\mu}{r}\frac{\partial \acute{H}_z}{\partial \phi}\right), \acute{E}_\phi=\frac{1}{g^2}\left(-\frac{\gamma}{r}\frac{\partial \acute{E}_z}{\partial \phi}+j\omega\mu\frac{\partial \acute{H}_z}{\partial r}\right), \qquad (9)$$

$$\acute{H}_r=\frac{1}{g^2}\left(\frac{j\omega\varepsilon}{r}\frac{\partial \acute{E}_z}{\partial \phi}-\gamma\frac{\partial \acute{H}_z}{\partial r}\right), \acute{H}_\phi=-\frac{1}{g^2}\left(j\omega\varepsilon\frac{\partial \acute{E}_z}{\partial r}+\frac{\gamma}{r}\frac{\partial \acute{H}_z}{\partial \phi}\right). \qquad (10)$$

The resulting coupling equations coincide with the analogous equations in the literature [2,3], but differ from the coupling equations (13), which were obtained in the monograph by D. Marcuse [4], since the equations for the longitudinal components in it are written in the form (12):

$$\acute{E}_z=E_z e^{-j\gamma z}e^{j\omega t}, \acute{H}_z=H_z e^{-j\gamma z}e^{j\omega t}, \qquad (11)$$

$$\acute{E}_r=-\frac{j}{g^2}\left(\gamma\frac{\partial \acute{E}_z}{\partial r}+\frac{\omega\mu}{r}\frac{\partial \acute{H}_z}{\partial \varphi}\right), \acute{E}_\varphi=-\frac{j}{g^2}\left(\frac{\gamma}{r}\frac{\partial \acute{E}_z}{\partial \varphi}-\omega\mu\frac{\partial \acute{H}_z}{\partial r}\right), \qquad (12)$$

$$\acute{H}_r=-\frac{j}{g^2}\left(-\omega\varepsilon\frac{\partial \acute{E}_z}{\partial \phi}+\gamma\frac{\partial \acute{H}_z}{\partial r}\right), \acute{H}_\varphi=-\frac{j}{g^2}\left(\frac{\gamma}{r}\frac{\partial \acute{H}_z}{\partial \varphi}+\omega\varepsilon\frac{\partial \acute{E}_z}{\partial r}\right). \qquad (13)$$

Definition of electromagnetic field (emf) components in the core and cladding of fiber optics

For internal problem just following conditions are valid: 0 ≤ r ≤ a, at r → 0 functions Jm (gr→0) are of finite significance (Fig.2a), Nm (gr→0) → - ∞, (Fig.2b), Hm(1) (gr→0) → - ∞, Hm(2) (gr→0) → + ∞.)

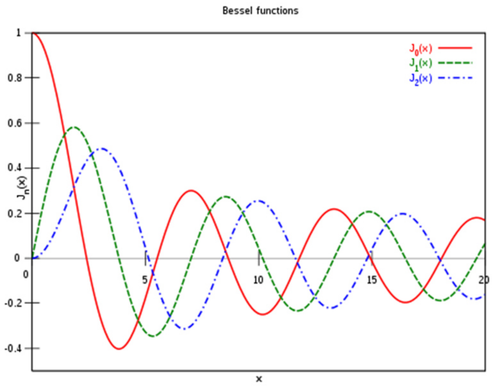

Fig. 2a. Bessel functions

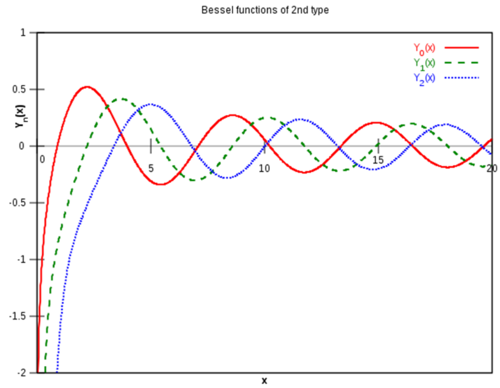

Fig. 2b. Bessel functions of 2nd type

Solutions of the external problem may be written down in form:

$$\left.\begin{aligned} \dot{E}'_z(r,\phi,z) &= \dot{A}_m \cdot J_m(g_1 r) \cdot e^{-\gamma z} \cdot e^{jm\phi} \\ \dot{H}'_z(r,\phi,z) &= \dot{B}_m \cdot J_m(g_1 r) e^{-\gamma z} e^{jm\phi} \end{aligned}\right\}, \tag{14}$$

where Am and Bm - unknown complex integral constants. From coupling equations other components are determined:

$$\dot{E}'_{r_1} = -\frac{1}{g_1^2}\left[\gamma g_1 \dot{A}_m J'_m(g_1 r) - \frac{\omega\mu_1 m}{r}\dot{B}_m J_m(g_1 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z}, \tag{15}$$

$$\dot{E}'_{\phi_1} = \frac{1}{g_1^2}\left[-\frac{jm\gamma}{r}\dot{A}'_m J_m(g_1 r) + j\omega\mu_1 g_1 \dot{B}'_m \overline{J_m}(g_1 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z}, \tag{16}$$

$$\dot{H}'_{r_1} = -\frac{1}{g_1^2}\left[\frac{\omega\varepsilon_1 m}{r}\dot{A}_m J_m(g_1 r) + \gamma g_1 \dot{B}_m \overline{J_m}(g_1 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z}, \tag{17}$$

$$\dot{H}'_{\phi 1} = -\frac{1}{g_1^2}\left[j\omega\varepsilon_1 g_1 \dot{A}_m \overline{J_m}(g_1 r) + \frac{j\gamma m}{r}\dot{B}'_m J_m(g_1 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z}, \tag{18}$$

where $J'_m(g_1 r) = \frac{\partial J_m(g_1 r)}{\partial r}$- derivative from Bessel's function,

$$(g_1 r)^2 = (k^2{}_1 + \gamma^2)r^2 = (\omega^2 \varepsilon_1 \mu_1 + \gamma^2)r^2 \; . \tag{19}$$

External problem solution (EMF in cladding)

If r → ∞ - the functions Jm(∞) → 0, Nm (r→∞) → 0, Hm(1) (r→∞) - , Hm(2) (r→∞) - finite significance functions. There are the following conditions for external problem: a ≤ r ≤ b.

For the electromagnetic field in the cladding, it is expedient to use solutions and to estimate the changes of the Hankel functions of the first and second kind from the radial coordinate at the corresponding propagation coefficient and the behavior of the function on the complex surface:

1) For a ≤ r < b, the Hankel function of the first kind increases from the core boundary to the perimeter of the cladding Hm(1)(k2r),

2) For a ≤ r < b, the Hankel function of the second kind, on the contrary, decreases with increasing radius Hm (2)(kr) if it is assumed that the transverse wave propagation coefficient in the shell is a complex value in the form: k2= - jg2.

Thus, from the standpoint of physical reliability, it is expedient to save the function Hm(2) (k2r)= Hm(2 (- jg2r) in the equation and write the z-projections of the vectors E and H on axes of reference in form:

$$\left.\begin{array}{l}\dot{E}_{z2}(r,\phi,z) = \dot{C}_m \cdot {H_m}^{(2)}(-jg_2 r) \cdot e^{jm\phi} \cdot e^{-\gamma z} \\ \dot{H}_{z2}(r,\phi,z) = \dot{D}_m \cdot {H_m}^{(2)}(-jg_2 r) \cdot e^{jm\phi} \cdot e^{-\gamma z}\end{array}\right\}, \tag{20}$$

where $\dot{C}_m, \dot{D}_m$ – unknown complex integral constants, $(-jg_2 \cdot r)^2 = ({k_2}^2 + \gamma^2) r^2 = [\omega^2 \varepsilon_2 \mu_2 + \gamma^2] \cdot r^2$, the' squared transverse coefficient of EMW propagation in the cladding along the coordinate r.

Other components are determinable from coupling equations:

$$\dot{E}_{r2} = -\frac{1}{(-jg_2)^2}\left[j\gamma(-jg_2)\cdot \dot{C}_m \cdot {H_m}^{(2)/}(-jg_2 r) - \frac{\omega\mu_2 m}{r}\cdot \dot{D}_m \cdot {H_m}^{(2)}(-jg_2 r)\right]\cdot e^{jm\varphi}\cdot e^{-\gamma z} =$$

$$= \frac{1}{{g_2}^2}\left[-j\gamma g_2 \cdot \dot{C}_m \cdot {H_m}^{(2)/}(-jg_2 r) - \frac{\omega\mu_2 m}{r}\cdot \dot{D}_m \cdot {H_m}^{(2)}(-jg_2 r)\right]\cdot e^{jm\varphi}\cdot e^{-\gamma z}, \tag{21}$$

$$\dot{E}_{\phi 2} = \frac{1}{(-jg_2)^2}\left[j\,\omega\mu_2(-jg_2)\cdot \dot{D}_m \cdot {H_m}^{(2)-}(-jg_2 r) - \frac{jm\gamma}{r}\cdot \dot{C}_m \cdot {H_m}^{(2)}(-jg_2 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z}, \tag{22}$$

$$\frac{1}{{g_2}^2}\left[-\omega\mu_2 g_2 \cdot \dot{D}_m \cdot {H_m}^{(2)-}(-jg_2 r) + \frac{jm\gamma}{r}\cdot \dot{C}_m \cdot {H_m}^{(2)}(-jg_2 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z},$$

$$\dot{H}_{r2} = \frac{1}{(-jg_2)^2}\left[\frac{j\omega\varepsilon_2 jm}{r}\cdot \dot{C}_m \cdot H_m^{(2)}(-jg_2 r) - j\gamma g_2 \cdot \dot{D}_m \cdot H_m^{(2)'}(-jg_2 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z} = \tag{23}$$

$$= \frac{1}{{g_2}^2}\left[\frac{\omega\varepsilon_2 m}{r}\cdot \dot{C}_m \cdot {H_m}^{(2)'}(-jg_2 r) + jg_2\gamma \cdot \dot{D}_m \cdot {H_m}^{(2)}(-jg_2 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z},$$

$$\dot{H}_{\phi 2} = -\frac{1}{(-jg_2)^2}\left[j\omega\varepsilon_2(-jg_2)\cdot \dot{C}_m \cdot {H_m}^{(2)'}(-jg_2 r) + \frac{j\gamma m}{r}\cdot \dot{D}_m \cdot {H_m}^{(2)}(-jg_2 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z} = \frac{1}{{g_2}^2}\left[\omega\varepsilon_2 g_2 \cdot \dot{C}_m \cdot {H_m}^{(2)'}(-jg_2 r) + \frac{j\gamma m}{r}\cdot \dot{D}_m \cdot {H_m}^{(2)}(-jg_2 r)\right]\cdot e^{jm\phi}\cdot e^{-\gamma z}, \tag{24}$$

where

$${H_m}^{(2)}(-jg_2 r) = \frac{\partial {H_m}^{(2)}(-jg_2 r)}{\partial r} = \frac{m}{-jg_2 r}{H_m}^{(2)}(-jg_2 r) - {H_{m+1}}^{(2)}(-jg_2 r), \tag{25}$$

- derivative from Hankel's function of the second kind.

The definition of unknown integral constants and deriving of the eigenvalue equation.

For the definition of unknown integrated constants, it is necessary to use the boundary conditions: at r = a all tangential projections of vectors E and H :

$$E'_{z1} = E'_{z2}, E'_{\phi 1} = E'_{\phi 2}, H'_{z1} = H'_{z2}, H'_{\phi 1} = H'_{\phi 2}, \tag{26}$$

are continuous (normal projections of vectors E and H – undergo sudden change):

1) $E'_{z1}(a) = E'_{z2}(a)$ and

$$A'_m \cdot J_m(g_1 a) e^{-\gamma z} \cdot e^{jm\phi} = C'_m \cdot H_m{}^{(2)}(-jg_2 a) e^{-\gamma z} \cdot e^{jm\phi} \text{ then,} \tag{27}$$

$$C'_m = A'_m \frac{J_m(g_1 a)}{H_m^{(2)}(-jg_2 a)}, \tag{28}$$

2) $H'_{z1}(a) = H'_{z2}(a)$ and

$$B'_m \cdot J_m(g_1 a) e^{-\gamma z} \cdot e^{jm\phi} = D'_m \cdot H_m{}^{(2)}(-jg_2 a) e^{-\gamma z} \cdot e^{jm\phi} \text{ then,} \tag{29}$$

$$D'_m = B'_m \frac{J_m(g_1 a)}{H_m^{(2)}(-jg_2 a)}, \tag{30}$$

3)
$$\frac{1}{g_1^2}\left[-\frac{jm\gamma}{a} A'_m J_m(g_1 a) + j\omega\mu_1 g_1 B'_m \overline{J_m}(g_1 a)\right] = \frac{1}{g_2{}^2}\left[-\omega\mu_2 g_2 \cdot D'_m \cdot H_m{}^{(2)-}(-jg_2 a) + \frac{jm\gamma}{a} \cdot C'_m \cdot H_m{}^{(2)}(-jg_2 a)\right]$$
$$\left[-\frac{\omega\mu_2}{g_2} \cdot B'_m \cdot \frac{J_m(g_1 a)}{H_m{}^{(2)}(-jg_2 a)} \cdot H_m{}^{(2)-}(-jg_2 a) + \frac{jm\gamma}{g_2{}^2 a} \cdot A'_m \cdot \frac{J_m(g_1 a)}{H_m{}^{(2)}(-jg_2 a)} H_m{}^{(2)}(-jg_2 a)\right] \text{ then,} \tag{31}$$

$$A'_m = B'_m \frac{\frac{\omega\mu_1}{(g_1 a)} \frac{J'_m(g_1 a)}{J_m(g_1 a)} - \frac{\omega\mu_2}{(-jg_2 a)} \frac{H^{(2)'}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}}{\frac{\gamma m}{a^2}\left(\frac{1}{g_1^2} - \frac{1}{(-jg_2)^2}\right)} \tag{32}$$

.

4)
$$H'_{\phi 1}(a) = H'_{\phi 2}(a) - \frac{1}{g_1^2}\left[j\omega\varepsilon_1 g_1 \dot{A}_m \overline{J_m}(g_1 a) + \frac{j\gamma m}{a} B'_m J_m(g_1 a)\right] =$$
$$= \frac{1}{g_2{}^2}\left[\omega\varepsilon_2 g_2 \cdot C'_m \cdot H_m{}^{(2)'}(-jg_2 a) + \frac{j\gamma m}{a} \cdot D'_m \cdot H_m{}^{(2)}(-jg_2 a)\right] =$$
$$= \left[\frac{\omega\varepsilon_2}{g_2} \cdot A'_m \frac{J_m(g_1 a)}{H_m^{(2)}(-jg_2 a)} \cdot H_m{}^{(2)'}(-jg_2 a) + \frac{1}{g_2{}^2}\frac{j\gamma m}{a} \cdot B'_m \frac{J'_m(g_1 a)}{H_m^{(2)}(-jg_2 a)} \cdot H_m{}^{(2)}(-jg_2 a)\right] \tag{33}$$

$$\acute{A}_m = \acute{B}_m \frac{\frac{\gamma m}{a}\left(\frac{1}{g_1^2} - \frac{1}{(-jg_2)^2}\right)}{-\frac{\omega\varepsilon_1}{g_1}\frac{J'_m(g_1 a)}{J_m(g_1 a)} + \frac{\omega\varepsilon_2}{(-jg_2)}\frac{H^{(2)'}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}}, \tag{34}$$

then

$\acute{A}_m \acute{B}_m C_m D_m - 4$ unknown integral constants may be determined from equations system.

Solving the equality with respect to $\acute{A}_m$ and $\acute{B}_m$:

$$\frac{\frac{\omega\mu_1}{(g_1 a)}\frac{J'_m(g_1 a)}{J_m(g_1 a)} - \frac{\omega\mu_2}{(-jg_2 a)}\frac{H^{(2)'}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}}{\frac{\gamma m}{a^2}\left(\frac{1}{g_1^2} - \frac{1}{(-jg_2)^2}\right)} = \frac{\frac{\gamma m}{a^2}\left(\frac{1}{g_1^2} - \frac{1}{(-jg_2)^2}\right)}{-\frac{\omega\varepsilon_1}{g_1 a}\frac{J'_m(g_1 a)}{J_m(g_1 a)} + \frac{\omega\varepsilon_2}{(-jg_2 a)}\frac{H^{(2)'}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}}, \tag{35}$$

we obtain the transcendental characteristics equation [so-called – eigenvalues equation (EVE)] in the form:

$$\left[-\frac{\omega\varepsilon_1}{g_1 a}\frac{J'_m(g_1 a)}{J_m(g_1 a)} + \frac{\omega\varepsilon_2}{(-jg_2 a)}\frac{H^{(2)\prime}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}\right] \times \times \left[\frac{\omega\mu_1}{g_1 a}\frac{J'_m(g_1 a)}{J_m(g_1 a)} - \frac{\omega\mu_2}{(-jg_2 a)}\frac{H^{(2)\prime}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}\right] = \frac{\gamma^2 m^2}{a^4}\left(\frac{1}{{g_1}^2} - \frac{1}{(-jg_2)^2}\right)^2. \tag{36}$$

Eigenvalue Equation (36) permits to find unknown parameter: γ - longitudinal propagation coefficient, which depends on transverse propagation coefficients g1 and g2 at different m – values and other parameters of this equation, i.e. on the structure of electromagnetic field in the core and cladding, parameters of higher types waves and basic performances of fiber optics. Equation (36) is solved only by numerical methods. The Eigenvalue Equation generally has a series of solutions, each of which corresponds to a definite structure of an electromagnetic field, called mode or wave type. The roots of this equation γmn = γmn (ω, g1, g2) are the propagation coefficients for electromagnetic waves of different structures in optical fiber, with the corresponding values of g1, g2, a.

The eigenvalue equations (EVE) as obtained in [1-7] are incorporated into table 1. And retain their comparative analysis. The general difference between the Eigenvalue Equations (36) in papers [1,2] and (37-41) [3-6] is the use of the second-kind Hankel function, which correctly corresponds to the physical decrease processes in components of the hybrid EMW propagating from the core to the boundary of the shell surface:

1) The right-hand sides Eigenvalue Equation (EVE) obtained in work [3] coincide with the EVE (36), and the left-hand side, in which the signs (+) appear in front of the first and second factors, respectively, are different from (36).

2) It should be noted that in [4] the problem was solved for a cylindrical dielectric waveguide without cladding (ε2=1). Comparison of EVE of work [4] (38 27) with EVE (36) shows: the difference between the right-hand side, expressed in the appearance of the sign (-), the difference between the left-hand sides in the signs and the absence of the complex factor j in front of the second terms of the expansion.

3) In Marcuse's book [5], in which a completely identical problem is solved, as a result of introducing the term e-jγz, instead of e-γz , the difference between EVE (36) and EVE (39) of work [5] is different: in the right-hand side there is no square of degree (apparently a misprint),In the left part – the difference in the sign (-) before the first member.

4) n work of [6], EVE in the form (40) is given, which can be transformed into Eq. (41) and, at the same time, estimates of the differences from EVE (36): in the right-hand side there is no square of degree, in the left part – the difference in the signs (-) and (+).

Analysis of the transcendental eigenvalue equation for hybrid electromagnetic waves which propagated in a regular stepped fiber optical line of communication

The equation (36) implies dependence) γmn = γmn (λ, m, g1a, g2a) - the coefficient of propagation of electromagnetic waves of various structures along the optical fiber at the corresponding values of λ, g1a , g2a, m. It should be noted that the values of the parameter m for hybrid EMW can be: m = 1; 2; 3; 4; 5; 10, i.e. along the φ coordinate can fit from one to 10 full wavelengths:

$$\gamma_{mn}\{(g_1a),(g_2a),m\} = \frac{1}{m}\cdot\sqrt{\left[-\frac{\omega\varepsilon_1}{g_1a}\frac{J'_m(g_1a)}{J_m(g_1a)}+\frac{j\omega\varepsilon_2}{g_2a}\frac{H^{(2)\prime}_m(-jg_2a)}{H^{(2)}_m(-jg_2a)}\right]\times \times\left[\frac{\omega\mu_1}{g_1a}\frac{J'_m(g_1a)}{J_m(g_1a)}-\frac{j\omega\mu_2}{g_2a}\frac{H^{(2)\prime}_m(-jg_2a)}{H^{(2)}_m(-jg_2a)}\right]}\cdot\frac{1}{\left(\frac{1}{(g_1a)^2}+\frac{1}{(g_2a)^2}\right)}, \tag{42}$$

where, γmn- is the coefficient of longitudinal propagation of EMW, which is related to the transverse coefficients of propagation g1 and g2 at the operating frequency of the signal ω at various values of m and parameters of the core and shell.

Analysis of the eigenvalue equation

The eigenvalue equation (42) in the general case has a number of solutions (roots of the equation), each of which corresponds to a certain structure of the electromagnetic field, called the wave type or mode [1,2], i.e. the solution to this equation is to find the relationship between the discrete values of the longitudinal propagation coefficients γmn (at extremely low attenuation coefficients) and the transverse propagation coefficients g1 and g2 at the operating signal frequency ω (or at the operating wavelength (for example, λ = 1.31 or 1.55 µm) for the given parameters of the core (ε1,μ1) and the shell (ε2,μ2). The values of γmn are called the eigenvalues of the EVE problem: γmn = γmn (ω, m,g1a , g2a). In the general case, the solution of this transcendental equation can be obtained only in numerical form, and the solution is associated with the six component vectors of the electromagnetic fields and, in general, they cannot be divided into transverse electric (TE) and transverse magnetic modes (TM), t. e. In the general case, in the core and in the cladding of the optical fiber, a multitude of hybrid waves of the type HEnm and EHnm arise.

Table 1.
Egenvalue equations (eve)

1.	$\left[-\frac{\omega\varepsilon_1}{g_1}\frac{J'_m(g_1a)}{J_m(g_1a)}+\frac{j\omega\varepsilon_2}{g_2}\frac{H^{(2)\prime}{}_m(-jg_2a)}{H^{(2)}{}_m(-jg_2a)}\right]\times \times\left[\frac{\omega\mu_1}{g_1}\frac{J'_m(g_1a)}{J_m(g_1a)}-\frac{j\omega\mu_2}{g_2}\frac{H^{(2)\prime}{}_m(-jg_2a)}{H^{(2)}{}_m(-jg_2a)}\right]= =\frac{m^2\gamma^2}{a^2}\left(\frac{1}{g_1{}^2}+\frac{1}{g_2{}^2}\right)^2.$	[2] (36)
2.	$\left[\frac{\omega\varepsilon_1}{g_1}\frac{J_m{}'(g_1a)}{J_m(g_1a)}+\frac{j\omega\varepsilon_2}{g_2}\frac{H_m^{(1)'}(jg_2a)}{H_m^{(1)}(jg_2a)}\right]* *\left[\frac{\omega\mu_1}{g_1}\frac{J_m{}'(g_1a)}{J_m(g_1a)}+\frac{j\omega\mu_2}{g_2}\frac{H_m^{(1)'}(jg_2a)}{H_m^{(1)}(jg_2a)}\right]=\frac{m^2\beta^2}{a^2}\left[\frac{1}{g_1^2}+\frac{1}{g_2^2}\right]^2.$	[3] (37)
3.	$\left[\frac{\omega\varepsilon_1}{g_1}\frac{J_m{}'(g_1a)}{J_m(g_1a)}-\frac{\omega\varepsilon_2}{g_2}\frac{H_m^{(1)'}(g_2a)}{H_m^{(1)}(g_2a)}\right]* *\left[\frac{\omega\mu_1}{g_1}\frac{J_m{}'(g_1a)}{J_m(g_1a)}-\frac{\omega\mu_2}{g_2}\frac{H_m^{(1)'}(g_2a)}{H_m^{(1)}(g_2a)}\right]=\frac{m^2\beta^2}{a^2}\left[\frac{1}{g_1^2}-\frac{1}{g_2^2}\right]^2.$	[4] (38)
4.	$\left[\frac{\omega\varepsilon_1}{g_1}\frac{J_m{}'(g_1a)}{J_m(g_1a)}+\frac{j\omega\varepsilon_2}{g_2}\frac{H_m^{(1)'}(jg_2a)}{H_m^{(1)}(jg_2a)}\right]* *\left[\frac{\omega\mu_1}{g_1}\frac{J_m{}'(g_1a)}{J_m(g_1a)}-\frac{j\omega\mu_2}{g_2}\frac{H_m^{(1)'}(jg_2a)}{H_m^{(1)}(jg_2a)}\right]=\frac{m^2\beta^2}{a^2}\left[\frac{1}{g_1^2}+\frac{1}{g_2^2}\right]^?.$	[5] (39)

5.	$\frac{k_0^2}{\beta^2}\left[\frac{1}{\chi r}\frac{I_n'(\chi r)}{I_n(\chi r)} - \frac{1}{i\upsilon r}\frac{H_n^{(1)'}(i\nu r)}{H_n^{(1)}(i\nu r)}\right]\cdot\left[\frac{n_1^2}{\chi r}\frac{I_n'(\chi r)}{I_n(\chi r)} - \frac{n_2^2}{i\upsilon r}\frac{H_n^{(1)'}(i\nu r)}{H_n^{(1)}(i\nu r)}\right] =$ $= n^2\left[\frac{1}{(\chi r)^2}+\frac{1}{(\nu r)^2}\right].$ (40) or $\left[\frac{1}{g_1}\frac{J_m'(g_1 a)}{J_m(g_1 a)} + \frac{j}{g_2}\frac{H_m^{(1)'}(jg_2 a)}{H_m^{(1)}(jg_2 a)}\right]\left[\frac{n_1^2}{g_1}\frac{J_m'(g_1 a)}{J_m(g_1 a)} + \frac{jn_2^2}{g_2}\frac{H_m^{(1)'}(jg_2 a)}{H_m^{(1)}(jg_2 a)}\right] =$ $= \frac{m^2\beta^2}{a^2}\left[\frac{1}{g_1^2 a^2}+\frac{1}{g_2^2 a^2}\right]^{?}$	[6] (41)

For symmetric electromagnetic waves, in which the electromagnetic field does not depend on the azimuth (φ -th) angle, for m = 0, the right-hand side of the characteristic equation (36 30) is zero and equation (42) splits into two relatively simple equations:

- for transverse electric wave types:

$$TE\ on(\ Hon) - modes - \left[-\frac{\omega\varepsilon_1}{g_1 a}\frac{J'_m(g_1 a)}{J_m(g_1 a)} + \frac{j\omega\varepsilon_2}{g_2 a}\frac{H^{(2)\prime}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}\right] = 0, \qquad (43)$$

-for transverse magnetic waves types:

$$THon = Tmon(E0n) - modes - \left[\frac{\omega\mu_1}{g_1 a}\frac{J'_m(g_1 a)}{J_m(g_1 a)} - \frac{j\omega\mu_2}{g_2 a}\frac{H^{(2)\prime}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}\right] = 0. \qquad (44)$$

Given that the derivatives of cylindrical functions

$: Z'_0(x) = -Z_1(x)$,we get :$J'_0(g_1 a) = J_1(g_1 a),$ (45)

and

$$H_0{}^{(2)\prime(-jg_2 r)} = \frac{\partial H_0{}^{(2)}(-jg_2 r)}{\partial r} = jg_2 \cdot H_1{}^{(2)}(-jg_2 r). \qquad (46)$$

Solutions of these equations can be approximately found:

1) For transverse electric modes that satisfy equation (44), from the boundary condition Hφ1= Hφ2, an expression follows that defines the relationship between the integral coefficients Bm, and Am [1], which are given in [1,2]: m = 0, then Bm →∞, and for Bm to be finite, it is necessary to satisfy the condition Am = 0, i.e. in this case, the projection Ez tends to zero, which means that transverse electric TE on (Hon) modes propagate in the dielectric waveguide.

Consider an approximate solution of equation (44), converting it to the form:

$$j\frac{\varepsilon_1 \cdot g_2}{\varepsilon_2 \cdot g_1}\frac{J'_m(g_1 a)}{J_m(g_1 a)} = \frac{H^{(2)\prime}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}. \qquad (47)$$

For small values of the transverse propagation coefficient g2 in the shell (g2a→0 or g2a <<1), the Hankel functions of the second kind of the ν - th order are written as an expansion:

$$H_\nu{}^{(2)}(-jg_2a) = j\frac{(\nu-1)!}{\pi}\cdot[\frac{2}{-jg_2a}]^\nu, \text{ for } \nu = 1,2,3\ldots \quad (48)$$

For the first order ν=1:

$$H_1{}^{(2)}(-jg_2a) = j\frac{1}{\pi}\cdot\left[\frac{2}{-jg_2a}\right], \quad (49)$$

a zero order –

$$H_0{}^{(2)}(-jg_2a) = j\frac{2}{\pi}\cdot ln[\frac{2}{Gg_2a}], \quad (50)$$

where number G=1,78 with the derivative

$$H_0{}^{(2)}{}^{'(-jg_2r)} = jg_2\cdot H_1{}^{(2)}(-jg_2r). \quad (51)$$

Therefore, the right side of equation (45) will be:

$$j\frac{\varepsilon_1\cdot g_2}{\varepsilon_2\cdot g_1}\frac{J'_m(g_1a)}{J_m(g_1a)} = \frac{H^{(2)}{}'_m(-jg_2a)}{H^{(2)}{}_m(-jg_2a)}. \quad (52)$$

And substituting in it the Hankel functions of the first and zero order, we get:

$$\frac{H_1{}^{(2)}(-jg_2a)}{H_0{}^{(2)}(-jg_2a)}) = \frac{j\frac{1}{\pi}\left[\frac{2}{-jg_2a}\right]}{j\frac{2}{\pi}ln\left[\frac{2}{Gg_2a}\right]} = -\frac{1}{jg_2a}\cdot\frac{1}{ln[\frac{2}{Gg_2a}]}, \quad (53)$$

where factorial zero is taken equal to one.

Then equation (45) is written in the form:

$$j\frac{\varepsilon_1\cdot g_2}{\varepsilon_2\cdot g_1}\cdot\frac{J_1(g_1a)}{J_0(g_1a)} = -\frac{1}{jg_2a}\cdot\frac{1}{ln\left[\frac{1}{Gg_2a}\right]}. \quad (54)$$

Let's transform the equation (46) to the form:

$$\frac{\varepsilon_2\cdot g_1}{\varepsilon_{1.}}\cdot\frac{J_0(g_1a)}{J_1(g_1a)} = g_2{}^2\cdot a\cdot ln\left[\frac{1}{Gg_2a}\right]. \quad (55)$$

For the right side of equation (47) , as g2a→0 tends, the magnitude of g22a tends to zero faster than $ln[\frac{1}{jg_2aG}]$ infinity. Therefore, for g2a→0, the right-hand side of equation (47) tends to zero, which means its approximate solution is written: $J_0\left(g_{1_n}a\right)\approx 0,$ with the remaining finite values.

As is known (Fig. 3), the Bessel function of the first kind of zero order $J_0(x)$ has a discrete number of zeros:

$$X = \varsigma_{0n} = g_{1_n}a = 2{,}405(n=1)_{5{,}52(n=2)_{8{,}654(n=3)}}\cdots \quad (56)$$

Thus, the solutions of equation (42) with g2a<<1 are discrete and characterize the types of transverse electric waves:- for the TE01 mode: g1₊a = 2.405;- for the TE02 mode: g1₊a = 5.52;- for TE03 mode: g1₊a = 8.654; etc.

It should be noted that these solutions characterize the critical values of the parameters g1n·a, which determine the critical frequencies (cutoff frequencies), below which the electromagnetic modes cease to exist as a physical wave structure.

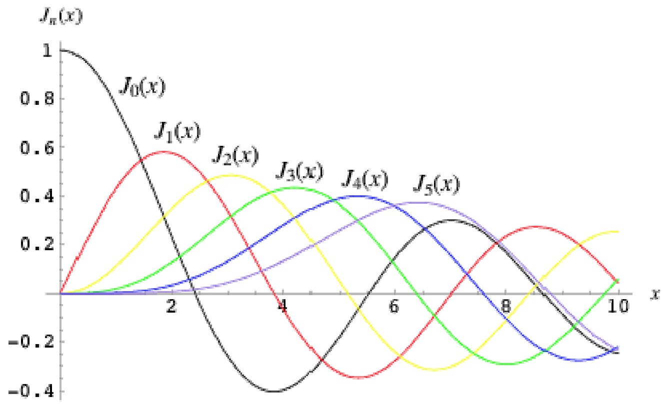

Fig.3. Graphs of changes in Bessel functions depending on the parameter x= g1n·a

For transverse magnetic modes that satisfy equation (44 33) , from the boundary condition Eφ1= Eφ2, , expression (57) follows, defining the relationship between the integral coefficients Bm and Am [1,2]:

$$\dot{A}_m = \dot{B}_m \frac{\dfrac{\omega\mu_1}{(g_1 a)}\dfrac{J'_m(g_1 a)}{J_m(g_1 a)} - \dfrac{\omega\mu_2}{(-jg_2 a)}\dfrac{H^{(2)'}{}_m(-jg_2 a)}{H^{(2)}{}_m(-jg_2 a)}}{\dfrac{\gamma m}{a^2}\left(\dfrac{1}{g_1^2} - \dfrac{1}{(-jg_2)^2}\right)} \tag{57}$$

If m = 0, then Bm = 0, if the denominator of the expression (57) is not equal to zero. From this it follows that at Bm = 0, the projection of Hz tends to zero, which means that the transverse magnetic TMon (Eon) modes propagate in the dielectric waveguide.

Let us determine the critical values of the parameters g1na at g2a→0 for the TMon - modes.

We write equation (44) with m = 0 in the form:

$$-j\frac{\mu_1 g_2}{\mu_2 g_1}\frac{J_1(g_1 a)}{J_0(g_1 a)} = \frac{H_1{}^{(2)}(-jg_2 a)}{H_0{}^{(2)}(-jg_2 a)}, \tag{58}$$

and use the expression for the TEon mode found above for the right-hand side with g2a→0, then

$$-j\frac{\mu_1 g_2}{\mu_2 g_1}\frac{J_1(g_1 a)}{J_0(g_1 a)} = -\frac{1}{jg_2 a}\cdot\frac{1}{ln\left[\frac{1}{Gg_2 a}\right]}. \tag{59}$$

Considering that μ1=μ2 =μ0·1 and transforming the formula (59), we get:

$$g_1\cdot\frac{J_0(g_1 a)}{J_1(g_1 a)} = jg_2{}^2\cdot a\cdot ln\left[\frac{1}{Gg_2 a}\right]. \tag{60}$$

Similar to equation (47), expression (60) is characterized by a discrete set of solutions (56) :

$$J_0\left(g_{1_n}a\right) \approx 0,\ \varsigma_{0n} = g_{1_n}a = 2{,}405(n=1)_{5{,}52(n=2)_{8{,}654(n=3)}},$$

and thus, TMon - modes have the same critical parameters as TEon - modes, i.e. and critical frequencies.

Symmetric electric E0n and magnetic H0n waves have circular symmetry (m = 0) (Fig.4).

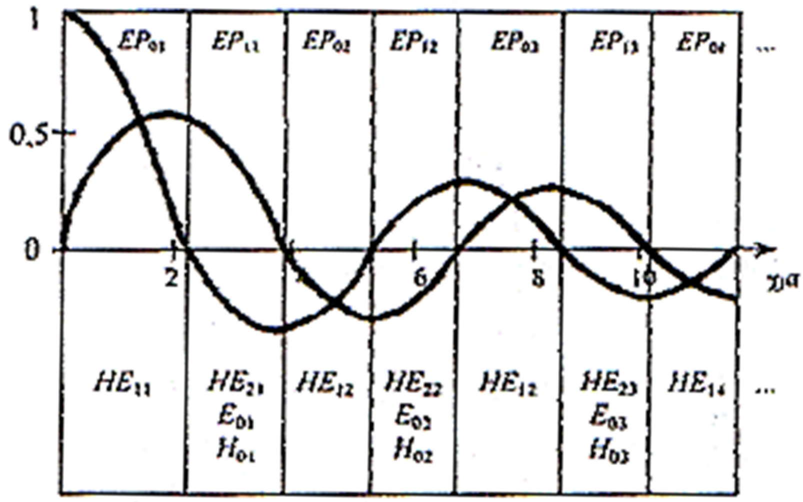

Fig.4. Distribution of hybrid modes depending on the number g1·a

Asymmetric electromagnetic waves satisfy the general boundary conditions $E'_{z1} = E'_{z2}, E'_{\phi1} = E'_{\phi2}, H'_{z1} = H'_{z2}, H'_{\phi1} = H'_{\phi2}$, the equation of eigenvalues EVE (36) is found for them and, therefore, in an optical fiber, electromagnetic waves can exist in the form of hybrid waves, having simultaneously with the transverse components also longitudinal components Ez and Hz.

If we perform numerical calculations of EVE with a change in gmn· a =ςmn, when n and m change 1.2.3, etc. in this case, gmn·a varies from 0 to ∞, one can obtain an infinite set of hybrid modes of the HEnm and EHnm type. As an example, for hybrid modes of the EHnm and EHnm type, we consider an asymmetric low-order mode of the HE11 type (m = 1, n = 1). The condition for solving the eigenvalue equation (4.8), i.e. the definition of the root of this equation is written in the form: g11·a =0 (single-mode regime), while the hybrid wave HE11 is a superposition of two types of waves E11 and H11. This wave has no critical frequency, i.e. can propagate along the fiber at all frequencies and at a certain ratio of the core diameter and the working wavelength.

For higher order modes, the roots of equation (36) will be:gmn· a =ςmn, for m = 1,2,3 ... the parameter ςmn is the n-th root of the equation: Jm(ςmn,)=0, at ςmn ≠ 0 .

Determination of critical frequencies (cutoff frequencies)/

An important parameter of any mode is the critical frequency (or cut-off frequency). As noted above, the mode ceases to exist as a physical wave structure, when the components of the electromagnetic field no longer decrease with distance from the core. The degree of EMF reduction with increasing radius is determined by the transverse wave propagation coefficient in the g2 shell. The m-th-order Hankel function of the second kind ${H_m}^{(2)}(-jg_2r)$ exponentially decreases to the periphery of the cladding with increasing values of the argument $(-jg_2r)$. For large values of the argument $(g_2r) \gg 1$, the asymptotic approximation ${H_m}^{(2)}(-jg_2r)$ is written in the form:

$$\begin{aligned} {H_m}^{(2)}(-jg_2r) &\approx \sqrt{\frac{2}{-j\pi g_2r}} \cdot e^{-\left[-jg2r-j\left(m+\frac{1}{2}\right)\frac{\pi}{2}\right]} \\ &= \sqrt{\frac{2}{-j\pi g_2r}} \cdot e^{-g_2r} \cdot e^{j\left(m+\frac{1}{2}\right)\frac{\pi}{2}}. \end{aligned} \qquad (61)$$

Thus, the second-kind Hankel function of the m-th order${H_m}^{(2)}(-jg_2r)$, in depending from the distance from the boundary core-cladding –of the optical fiber changes according to the law:

$${H_m}^{(2)}(-jg_2r) \Rightarrow \frac{e^{-g_2r}}{\sqrt{-jg_2r}}. \qquad (62)$$

That is, the numerator decreases faster than the denominator. With a decrease g_2, the electromagnetic field begins to be redistributed and at g_2= 0 in the optical fiber the electromagnetic waves will not propagate, i.e. there is a so-called critical mode. It is the wavelength (or frequency) at which this occurs is called the critical wavelength or frequency (cutoff frequency).

Thus, the condition of the critical mode is the ratio:

$$(-jg_2 \cdot r)^2 = ({k_2}^2 + {\gamma_n}^2)r^2 = [\omega^2\varepsilon_2\mu_2 + {\gamma_n}^2] \cdot r^2 = 0, \qquad (63)$$

$$g_2 = \sqrt{{\gamma_n}^2 + {k_2}^2} = 0 \qquad (64)$$

That is ${\gamma_{kr}}^2(\lambda = \lambda_{kr}) = -{k_2}^2 = \pm j\omega_{kr}\sqrt{\varepsilon_2\mu_2}$ .Express the critical frequency in terms of the parameters of the fiber core from the equation:

$$(g_1r)^2 = (k^2{}_1 + \gamma^2)r^2 = (\omega^2\varepsilon_1\mu_1 + \gamma^2)r^2 \qquad (65)$$

Then:

$$\begin{aligned} {g_{1n}}^2 = {k_{1n}}^2 + {\gamma_n}^2 = {k_{1n}}^2 - {k_{2kr}}^2 = {\omega_{kr}}^2(\varepsilon_1\mu_0) - {\omega_{kr}}^2(\varepsilon_2\mu_0) = \\ = {\omega_{kr}}^2(\varepsilon_1 - \varepsilon_2)\varepsilon_0\mu_0 = (\frac{2\pi f_{cr}}{c})^2 \cdot (\varepsilon_1 - \varepsilon_2), \end{aligned} \qquad (66)$$

where the critical frequency is determined in the form:

$$f_{kr_n} = \frac{g_1 \cdot c}{2\pi} \cdot \frac{1}{\sqrt{{n_1}^2 - {n_2}^2}} = \frac{g_{1n} \cdot a}{2\pi a \cdot \sqrt{\varepsilon_0(\varepsilon_{r_1} - \varepsilon_{r_2})\mu_0}} = \frac{(g_{1_n} \cdot a) \cdot c}{\pi d \cdot \sqrt{\varepsilon_{r_1} - \varepsilon_{r_2}}}. \quad (67)$$

And critical wavelength:

$$\lambda_{kr_n} = \frac{\pi d \cdot \sqrt{\varepsilon_{r_1} - \varepsilon_{r_2}}}{g_{1_n} \cdot a} = \frac{\pi d \sqrt{{n_1}^2 - {n_2}^2}}{p_{1_n}}. \quad (68)$$

Thus, the critical frequency for the nth mode is determined by the parameter p1n=g1n·a for a given core radius and the difference in relative dielectric constant (ε1-ε2). When analyzing the eigenvalue equation, only a lower-order asymmetric mode with m = 1 and n = 1 can have a zero value g11a =0 and hence the critical frequency is zero. This mode, therefore, can propagate on any frequency. This regime of propagation of electromagnetic waves is called degenerate.

In the real case for the HE11 mode, the condition of propagation of the EMW over the parameter p1n=g1n·a cut-off condition can be found from the inequality 0 < (g1n a) < 2,405

The phase propagation velocity of the electromagnetic wave along the fiber cladding is determined from a simple expression:

$$v = \frac{\omega}{k_2} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_2 \mu_0 \mu_2}} = \frac{c}{\sqrt{\varepsilon_{r_2}}}, \quad (69)$$

at a frequency equal to the critical (when and at high frequencies, when almost all the energy is concentrated inside the core:

$$v = \frac{\omega}{k_1} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_1 \mu_0 \mu_1}} = \frac{c}{\sqrt{\varepsilon_1}}. \quad (70)$$

In accordance with the solution of the internal problem, an electromagnetic wave of type HE11 exists in the core in the form of two mutually orthogonal polarizations: vertical - HE11V and horizontal - HE11H, corresponding to the change in angle φ : ejmφ = cosφ +j·sinφ .

As an example, the values of the part of the roots g1n a a Bessel functions, depending on the order n and the roots of the Bessel functions, are given in tabl. 2.

Table 2.

Parameters of p1n=g1n·a for the number of the root function with different m

Wave type	Function order n	1	2	3
E0 n, H0n	0	2,405	5,520	8,654
HE11	1	0,000	3,832	7,016
EH1 n	1	3,832	7,016	10,173
HE2 n	2	3,050	5,538	8,665
EH2 n	2	5,136	8,417	11,620

As follows from the above, the Bessel functions of the first kind Jm(pmn)=0 of the n – th order give an infinite number of roots p1n =g1n ·a. Moreover, the roots of the function J0(p0n) determine the structure of the electromagnetic field of the symmetric waves E0n and H0n, and Jm(pmn) with n≠0 determine the structure of the asymmetric hybrid waves HE mn and EH mn. Separate propagation of asymmetric waves of type Emn and Hmn through the fiber is impossible. In the optical fiber, they exist together, i.e. the longitudinal components Ez ≠0 and Hz≠0 . It is these waves that are called hybrid waves and are designated HE mn if the field in cross section resembles a field of type H, or EH mn if the field in cross section resembles waves of type E. It should be noted that lasers, as coherent sources of optical radiation, create a linearly polarized wave at the input of the fiber, which leads to the excitation in the fiber of the primary waves unlike hybrid waves HEmn and EHmn . This problem can be considered by grouping the modes by the ranges of values pmn =gmn·a between the adjacent roots of the Bessel functions J0(pmn,) and J1(pmn,), as given in tfbl. 3.

Table 3.

Grouping the modes by the ranges of values pmn =gmn·a

Spacing between root values pmn=ςmn = gmn·a	Wave type (mode)
0-2,405	HE11, E0 n, H0n
2,405-3,832	H01, E01,HE21
3,832-5,136	HE12,EH11, HE31
5,136-5,520	EH21, HE42
5,520-6,380	H02, E02, HE22
6,380-7,016	EH31, HE51
7,016-7,588	HE13,EH12, HE32
7,588-8,417	EH41, HE61

The sums of the fields of the considered wave groups give the corresponding linearly polarized modes, denoted by LPmn, or the polarization modes, denoted by EPmn. EPmn waves with a single linear polarization represent only an approximation of the actual eigen waves HE mn and EH mn in the core of the fiber. Any LP1n wave consists of H0n and HE2n waves with one polarization and of E0n and HE2n waves with another polarization. The LPmn wave of higher order n around the circumference of the core consists of HE m+1,n and EH m-1,n. More precisely, of course, it is necessary to use the spectrum of the waves HEmn and EHmn. For weakly guiding fibers with refractive indices n1≈n2 , it is convenient to use LPmn waves. Only when LPmn waves propagate through a fiber over a long distance, do they split into their own HE and EH waves due to the difference in their eigenvalues (i.e., phase velocities). Therefore, when introducing a linearly polarized laser radiation into a stepped fiber, one observes an inconsistency between the eigenmodes HE mn and EHmn both in the field structure and in the polarization. This leads to a strong transformation of waves in the initial part of the fiber and, as a consequence, to an increase in the power loss of the propagating EMW due to "leakage" of modes into the shell.

The length of the establishment of the equilibrium state of the modes can be determined with the approximate formula:

$$L_m = \frac{d}{4\theta_{cr}} \cdot exp(\frac{\nu}{2}), \quad (71)$$

where Lm is the fiber length (in m), at which 90% of the power of all flowing modes is radiated, d is the core diameter (in m), θcr is the solid angle of aperture in radians, ν is a power parameter. For example, when d = 10 μm., θcr = 0.14 and ν = 30, the value of the length of the establishment of the equilibrium state of the modes will be Lm = 58 m.

Algorithm for calculation of optical parameters focl

The algorithm of the engineering method for calculating the optical parameters of a single-mode and multi-mode stepped fiber can be summarized in tabl. 4 [2,8].

Calculation of energy balance in a fiber optical communication line (FOCL)

Let a laser diode be used as a radiation source for transmitting signals through a single-mode fiber, and avalanche photodiodes as a detector at the receiving end. The output power of the source and the sensitivity of the detector depend on the required bit rate of digital information and the line length. In the general case, the power of the source is selected from the required line length, while the sensitivity of the detector should be the best.

The line is physically realizable when the following condition is met:

$$Pt – a > Pmin +\delta, \quad (72)$$

where Pt is the transmitted power level (dBm), a is the total operating attenuation in the line (dB), Pmin is the receiver sensitivity (dBm), δ is the energy margin (dB).

For line loss analysis, the energy balance is usually used, which is defined as the difference between the transmitted power and the receiver sensitivity, i.e., if the maximum transmitted power is - (-10) dBm, and the receiver sensitivity is (-30) dBm, then the energy the power balance will be equal to

$$B= Pt\ мак - Pmin =20\ dB. \quad (73)$$

The overall energy balance determines the permissible level of losses in the line from the terminal equipment to, for example, the first regenerator. If the total losses in the FOCL do not exceed 20 dB, then the conditions for stable reception are ensured.

When calculating the total losses in FOCL, the following main parameters are taken into account:

- line length L;
- coefficient of linear attenuation in optical fiber α;
- number of detachable connections i;
- average losses in the ai detachable connector ai;

• the number of non-detachable connectors n;
• average losses in the one-piece connector an;
• number of taps s;
• average losses in the coupler as;
• energy reserve δ (above the sensitivity level of the photodetector).

The considered algorithm can be used for engineering calculation of the main optical parameters of a single-mode and multimode stepped fiber of fiber-optic communication lines in the Mathcad program.

Typically, the energy margin δ is selected in the range from 3 to 6 dB. The total attenuation in the FOCL can be determined as follows:

$$a = \alpha \cdot L + i \cdot a_i + n \cdot a_n + s \cdot a_s - P_t - P_{rmin} - \delta = B - \delta, \quad (74)$$

where Pt is the power of the transmitter (laser), Prmin is the sensitivity of the photodetector, B is the energy balance.

From formula (73), taking into account the standard construction length of the cable lb, the maximum length of the regenerative transmission section can be calculated both in terms of attenuation and dispersion. For single-mode fibers that have good dispersion characteristics, the length of the regeneration section is determined by the total attenuation:

$$L_{ri} = \frac{B - i \cdot a_i - n \cdot a_n - s \cdot a_s - \delta}{\alpha + \frac{a_n}{l_b}}, \quad (75)$$

where B is the energy balance, lb is the construction length of the cable.

As an example, the following is the calculation of the maximum distance to the first regenerator of the main fiber-optic communication line with the following data:

$$P_{PAR} = 10dBm$$
$$P_{UZTmin}$$
$$a_{PAR} = -3_{dBm}$$
$$a_{SS} = 1dB$$
$$a_{NS} = 0.1dB$$
$$a_{SAZ} = 2dB$$
$$L_{CEL} = 2km$$
$$\alpha = \frac{0.25dB}{km}$$
$$L_{MAX} = \frac{10 - (-40) - 3 - 2 - 2 + 0.1 - 6}{0.25 + \frac{0.1}{2}} = 123{,}7\ km$$

For the railway line - Riga - Krustpils - Daugavpils in fig. 5. shows the power distribution along the path for the following data: $L_1 = 120$ km, $L_2 = 90$km, where M=1 is one coupler.

Table 4.
The algorithm of the engineering method for calculating the optical parameters

№	Parameter	Formula	Unit of measurement
1.	Refractive index of the coating n2:	$n_2 = n_1 - \Delta n \cdot 2n_1$	-
2.	Number aperture NA	$\mathrm{NA} = \sqrt{n_1^2 - n_2^2}$	-
3.	Characteristic (normalized) frequency	$\zeta = \frac{\pi d}{\lambda} \cdot NA$	-
4.	Critical wavelength	$\lambda_{\mathrm{kr}} = \frac{\pi d}{2{,}405} \cdot NA$	μm
5.	Linear coefficient of dielectric energy loss:αp , where tgδ is the dielectric loss tangent - tgδ = (10-12-10-10)	$\alpha_a = \frac{\pi n_1 \mathrm{tg}\delta}{\lambda} \cdot 8{,}69 \cdot 10^3$	dB/km
6.	Linear coefficient due to Rayleigh scattering losses: αi where KR is the Rayleigh scattering coefficient -KR = 0,8 dB·μm4/km	$\alpha_i = \frac{K_R}{\lambda^4}$	dB/km
7.	Approximate value of the total linear loss factor	$\alpha = \alpha_P + \alpha_R$	dB/km
8.	specific chromatic dispersion: M(λ) where B (λ) is the waveguide specific dispersion M (λ) is the material specific dispersion	$D(\lambda) = M(\lambda) + B(\lambda)$	πc/nm·km
9.	Time chromatic dispersion	$\tau = \Delta\lambda \cdot D(\lambda) \cdot L$	ns
10.	Approximate bandwidth 0,187 <k ≤ 1	ΔF≈k/т	MHz

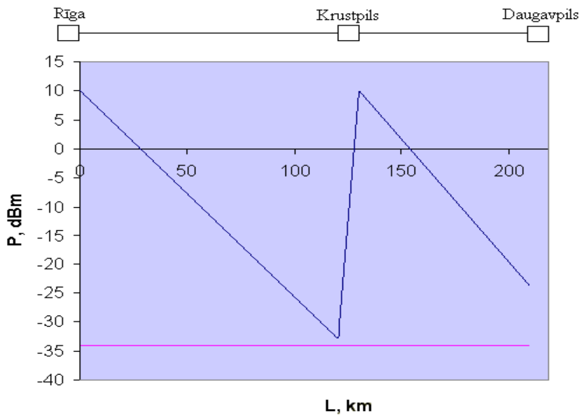

Fig. 5. Distribution of optical signal power along the main FOCL Riga - Krustpils - Daugavpils

Conclusions

Thus, the paper has solved the boundary value problem of EMW propagation in a regular stepped confocal cylindrical optical waveguide, as a result of which an eigenvalue equation (EVE) was obtained that describes the origin of an infinite number of electromagnetic hybrid modes.

The analysis of the eigenvalue equation showed the conditions for the occurrence of transverse-electric, transverse-magnetic and hybrid electromagnetic waves, and also estimated the conditions for single-mode and multi-mode regimes of EMW propagation. The critical lengths of hybrid EME are determined.

An algorithm for an engineering method for calculating the main optical parameters of FOCL is proposed, the main of which are: the linear attenuation coefficient, the bandwidth of the optical channel and the energy balance along the FOCL.

References

1. Popov V. Modern problems of physics. Fiber optics. / V. Popov .- Riga: LIR&CNX, 1991, -39 p.
2. Popov V. Foundations theory of guiding systems. / V. Popov.- Riga: RTU, 2017,- 206 p. (E-version).
3. Goldshtein L, Electromagnetic fields and waves. / L. Goldshtein, N. Zernov .- M.: Sov.radio, 1971,- 430p.
4. Volman V. Technical electrodynamics. / V. Volman, Ju. Pimenov .- M.: Sov.Radio, 1971,- 487 p.
5. Marcuse D. Light Transmission Optics. / D. Marcuse .- N.Y., 1972,- 289 p.
6. Ivanov A. Fiber optics. Components, communication systems, measuring. / A. Ivanov .- M.: Cyrus systems company, 1999,- 664 p.
7. Popovs V. Transporta sakaru līnijas. (Kabeļu sakari līniju parametru aprēķinu pamati). / V. Popovs.-/ Rīga: RTU Izdevniecība, 2008, -114 pp.